Planar spin-transfer device with a dynamic polarizer.

Ya. B. Bazaliy,[1] D. Olaosebikan,[2] and B. A Jones[1]

[1]*IBM Almaden Research Center, 650 Harry Road, San Jose, CA 95120*
[2]*Department of Physics, Cornell University, Ithaca, NY 14853*

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In planar nano-magnetic devices magnetization direction is kept close to a given plane by the large easy-plane magnetic anisotropy, for example by the shape anisotropy in a thin film. In this case magnetization shows effectively in-plane dynamics with only one angle required for its description. Moreover, the motion can become overdamped even for small values of Gilbert damping. We derive the equations of effective in-plane dynamics in the presence of spin-transfer torques. The simplifications achieved in the overdamped regime allow to study systems with several dynamic magnetic pieces ("free layers"). A transition from a spin-transfer device with a static polarizer to a device with two equivalent magnets is observed. When the size difference between the magnets is less than critical, the device does not exhibit switching, but goes directly into the "windmill" precession state.

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I. INTRODUCTION

The prediction[1,2] and first experimental observations[3,4,5,6,7,8] of spin-transfer torques opened a new field in magnetism which studies non-equilibrium magnetic interactions induced by electric current. Since such interactions are relatively significant only in very small structures, the topic is a part of nano-magnetism. The current-induced switching of magnetic devices achieved through spin-transfer torques is a candidate for being used as a writing process in magnetic random access memory (MRAM) devices. The MRAM memory cell is a typical example of a spintronic device in which the electron spin is used to achieve useful logic, memory or other operations normally performed by electronic circuits.

To produce the spin-transfer torques, electric currents have to flow through the spatially non-uniform magnetic configurations in which the variation of magnetization can be either continuous or abrupt. The first case is usually experimentally realized in magnetic domain walls.[3,9,10,11] Here we will be focusing on the second case realized in the artificially grown nano-structures. Such spin-transfer devices contain several magnetic pieces separated by non-magnetic metal spacers allowing for arbitrary angles between the magnetic moments of the pieces. Magnetization my vary within each piece as well, but that variation is usually much smaller and vanishes as the size of piece is reduced, or for larger values of spin-stiffness of magnetic material. The typical examples of a system with discrete variation of magnetization are the "nano-pillar" devices[8] (Fig.1A). Their behavior can be reasonably well approximated by assuming that magnetic pieces are mono-domain, each described by a single magnetization vector $\vec{M}(t) = M_s\vec{n}(t)$ where $\vec{n}$ is the unit vector and M_s is the saturation magnetization. The evolution of $\vec{n}(t)$ is governed by the Landau-Lifshitz-Gilbert (LLG) equation with spin-transfer terms.[2,12]

It is often the case that magnetic pieces in a spin-transfer device have a strong easy-plane anisotropy. For example, in nano-pillars both the polarizer and the free magnetic layer are disks with the diameter much larger than the thickness. Consequently, the shape anisotropy makes the plane of the disk an easy magnetic plane. In the planar devices[13] built from thin film layers (Fig. 1B) the shape anisotropy produces the same effect. When the easy-plane anisotropy energy is much larger then all other energies, the deviations of $\vec{n}(t)$ from the in-plane direction are very small. An approximation based on such smallness is possible and provides an effective description of the magnetic dynamics in terms of the direction of the projection of $\vec{n}(t)$ on the easy plane, i.e. in terms of one azimuthal angle. In this paper we derive the equations for effective in-plane motion in the presence of spin-transfer effect and discuss their use by considering several examples.

In the absence of spin-transfer effects the large easy-plane anisotropy creates a regime of overdamped motion even for the small values of Gilbert damping constant $\alpha \ll 1$.[14] In that regime the equations simplify further. Here the overdamped regime is discussed in the presence of electric current. The reduction of the number of equations allows for a simple consideration of a spin-transfer device with two dynamic magnetic pieces. We show how an asymmetry in the sizes of these pieces creates a transition between the polarizer-analyzer ("fixed layer - free layer") operation regime[2,8,12,15] and the regime of nearly identical pieces where current leads

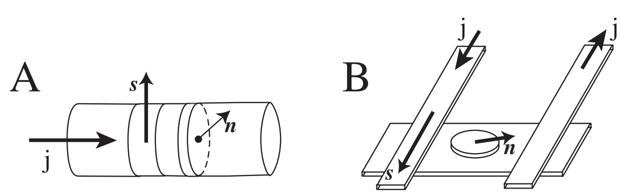

FIG. 1: Planar spin-transfer devices

not to switching, but directly to the Slonczewski "windmill" dynamic state.[2] Finally, we point out the limitations of the overdamped approximation in the presence of the spin-transfer torques.

II. DYNAMIC EQUATIONS IN THE LIMIT OF A LARGE EASY-PLANE ANISOTROPY

Magnetization dynamics in the presence of electric current is governed by the LLG equation with the spin-transfer term.[2,12] For each of the magnets in the device shown on Fig. 1A

$$\dot{\vec{n}} = \frac{\gamma}{M_s}\left[-\frac{\delta E}{\delta \vec{n}} \times \vec{n}\right] + u[\vec{n} \times [\vec{s} \times \vec{n}]] + \alpha[\vec{n} \times \dot{\vec{n}}] \quad (1)$$

where $\vec{s}(t)$ is the unit vector along the instantaneous magnetization of the other magnet and the spin-transfer magnitude

$$u = g(P)\frac{\gamma(\hbar/2)}{VM_s}\frac{I}{e} \quad (2)$$

is proportional to the electric current I. Here e is the (negative) electron charge, so u is positive when electrons flow into the magnet. Due to the inverse proportionality to the volume V, the larger magnets become less sensitive to the current and can serve as spin-polarizers with a fixed magnetization direction. As for the other parameters, γ is the gyromagnetic ratio, $g(P, (\vec{n} \cdot \vec{s}))$ is the Slonczewski spin polarization factor[2] which depends on many system parameters,[16,17] and α is the Gilbert damping which also depends on $\vec{n}$ and $\vec{s}$ when spin pumping[18] is taken into account. We will restrict our treatment to the constant g and α to focus on the effects specific to the strong easy plane anisotropy.

In terms of the polar angles (θ, ϕ) the LLG equation (1) has the form

$$\begin{aligned} \dot{\theta} + \alpha\dot{\phi}\sin\theta &= -\frac{\gamma}{M\sin\theta}\frac{\partial E}{\partial \phi} + u(\vec{s}\cdot\vec{e}_\theta) \\ \dot{\phi}\sin\theta - \alpha\dot{\theta} &= \frac{\gamma}{M}\frac{\partial E}{\partial \theta} + u(\vec{s}\cdot\vec{e}_\phi) \end{aligned} \quad (3)$$

where the tangent unit vectors $\vec{e}_\theta$ and $\vec{e}_\phi$ are defined in Appendix A.

We will consider a model for which the energy of a magnet is given by

$$E = \frac{K_\perp \cos^2\theta}{2} + E_r(\phi) \quad (4)$$

with $K_\perp$ being the easy-plane constant, E_r being the "residual" in-plane anisotropy energy and z-axis directed perpendicular to the easy plane. The limit of a strong easy-plane anisotropy is achieved when the maximal variation of the residual energy is small compared to the easy-plane energy, $\Delta E_r \ll K_\perp$. In this case $\theta = \pi/2 + \delta\theta$ with $\delta\theta \ll 1$.

To estimate $\delta\theta$, consider the motion of magnetization initially lying in-plane off the minimum of E_r and neglect for the moment the spin-transfer terms in Eq.(3). Magnetization starts moving and a certain deviation from the easy plane is developed. For the estimate, assume that the energy is conserved during this motion (the presence of damping will only decrease $\delta\theta$). Then

$$|\delta\theta| \sim \sqrt{\frac{\Delta E_r}{K_\perp}} \ll 1 \quad (5)$$

We can now linearize the right hand sides of equations (3) in small $\delta\theta$. On top of that, some terms on the left hand sides of (3) turn out to be small and can be discarded. Indeed, taking into account the smallness of α one gets the estimates

$$\begin{aligned} \dot{\theta} &\sim -\frac{\gamma}{M_s}\frac{\partial E_r}{\partial \phi} \sim -\frac{\gamma}{M_s}\Delta E_r \\ \dot{\phi} &\sim \frac{\gamma}{M_s}K_\perp\delta\theta \sim \frac{\gamma}{M_s}\sqrt{K_\perp \Delta E_r} \end{aligned}$$

Consequently $\dot{\theta} \sim \dot{\phi}\sqrt{\Delta E_r/K_\perp} \ll \dot{\phi}$ and $\dot{\phi} \gg \alpha\dot{\theta}$, therefore the second term on the left hand side of the second equation of the system (3) can be discarded. No simplification happens on the left hand side of the first equation, where $\dot{\theta}$ and $\alpha\dot{\phi}$ can be of the same order when $\alpha \lesssim \sqrt{\Delta E_r/K_\perp}$.

Putting the spin-transfer terms back we get the form of equations in the limit of large easy-plane anisotropy:

$$\begin{aligned} \dot{\delta\theta} + \alpha\dot{\phi} &= -\frac{\gamma}{M_s}\frac{\partial E}{\partial \phi} + u(\vec{s}\cdot\vec{e}_\theta) \\ \dot{\phi} &= \frac{\gamma K_\perp}{M_s}\delta\theta + u(\vec{s}\cdot\vec{e}_\phi) \end{aligned} \quad (6)$$

Expressions for the scalar products in (6) in terms of polar angles are given in Appendix A.

The second equation shows that $\delta\theta$ can be expressed through $(\phi, \dot{\phi})$. Small out-of-plane deviation becomes a "slave" of the in-plane motion.[14] We get

$$\frac{M_s}{\gamma K_\perp}\left(\ddot{\phi} - u\frac{d(\vec{s}\cdot\vec{e}_\phi)}{dt}\right) + \alpha_i\dot{\phi} = -\frac{\gamma}{M_s}\frac{\partial E_r}{\partial \phi} + u(\vec{s}\cdot\vec{e}_\theta) \quad (7)$$

The term with the second time derivative $\ddot{\phi}$ decreases with increasing $K_\perp$. As pointed out in Ref. 14, in the absence of spin-transfer this term can be neglected when $K_\perp > \Delta E_r/\alpha^2$. Mathematically this corresponds to a transition from an underdamped to an overdamped behavior of an oscillator as the oscillator mass decreases.

With spin-transfer terms the overdamped approximation gives an equation

$$\alpha\dot{\phi} - \xi\frac{d}{dt}(\vec{s}\cdot\vec{e}_\phi) = -\frac{\gamma}{M_s}\frac{\partial E_r}{\partial \phi} + u(\vec{s}\cdot\vec{e}_\theta) \quad (8)$$

where $\xi = uM_s/(\gamma K_\perp)$. The range of this equation's validity will be discussed in Sec. IV. The scalar products in Eq. (8) have to be expressed through the polar angles

$(\theta_s(t), \phi_s(t))$ of vector $\vec{s}$, and linearized with respect to $\delta\theta$ (see Appendix, Eq. A4), which is then substituted from Eq. (6). Finally, the equation is linearized with respect to small spin-transfer magnitude u. We get:

$$\alpha\dot{\phi} - \xi\left(\frac{d}{dt}\big[\sin\theta_s\sin(\phi_s-\phi)\big] - \sin\theta_s\cos(\phi_s-\phi)\dot{\phi}\right) = -\frac{\gamma}{M_s}\frac{\partial E_r}{\partial\phi} - u\cos\theta_s\ , \tag{9}$$

describing the in-plane overdamped motion of an analyzer with a polarizer pointed in the arbitrary direction. Next, we show how some known results on spin-transfer systems are recovered in the approximation (9).

Consider the device shown on Fig. 1A and assume that the first magnet is very large. As explained above, this magnet is not affected by the current and serves as a fixed source of spin-polarized electrons for the second magnet called the analyzer, or the "free" layer. The magnetization dynamics of the analyzer is described by Eq. (3). The case of static polarizer is extensively studied in the literature.

First, consider the case of *collinear switching*, experimentally realized in a nano-pillar device with the analyzer's and polarizer's easy axes along the $\hat{x}$ direction: $E_r = (1/2)K_{||}\sin^2\phi$, $\vec{s} = (1,0,0)$.[7] Using Eq. (9) with $\theta_s = \pi/2$, $\phi_s = 0$ we get

$$(\alpha + 2\xi\cos\phi)\dot{\phi} = -\frac{\gamma K_{||}}{2M_s}\sin 2\phi \tag{10}$$

Without the current, there are four possible equilibria of the analyzer. Two stable equilibria are the parallel ($\phi = 0$) and anti-parallel ($\phi = \pi$) states. Two perpendicular equilibria ($\phi = \pm\pi/2$) are unstable. Linearizing Eq. (10) near equilibria one finds solutions the form $\delta\phi(t) \sim \exp(\omega t)$ with eigenfrequencies

$$\begin{aligned}
\omega &= -\frac{\gamma K_{||}}{M_s(\alpha+2\xi)}, \qquad (\phi \approx 0)\\
\omega &= -\frac{\gamma K_{||}}{M_s(\alpha-2\xi)}, \qquad (\phi \approx \pi)\\
\omega &= \frac{\gamma K_{||}}{M_s\alpha}, \qquad (\phi \approx \pm\pi/2)
\end{aligned}$$

The equilibria are stable for $\omega < 0$ and unstable otherwise. Thus the parallel state is stable for $\xi > -\alpha/2$, the antiparallel state is stable for $\xi < \alpha/2$, and the perpendicular states cannot be stabilized by the current. These conclusions agree with the results of Refs. 2,7,12. The stability regions are shown in Fig. 2A.

Note how Eq. (10) emphasizes the fact that spin-transfer torque destabilizes the equilibria by making the effective damping constant $\alpha_{eff} = \alpha + 2\xi\cos\phi$ negative, while the equilibrium points remain a minimum of the energy E_r. Any appreciable influence of the current on the position and nature (minimum or maximum) of the equilibrium can only be observed at the current magnitudes $1/\alpha$ times larger than the actual switching current.[12]

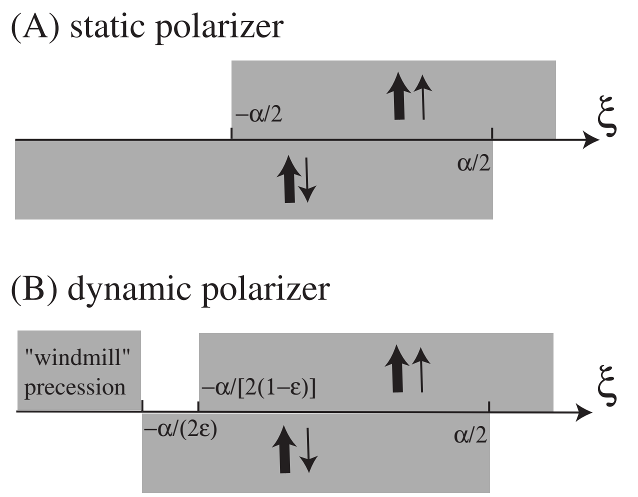

FIG. 2: Stability regions for systems with static (A) and dynamic (B) polarizers as a function of applied current, $\xi = g(P)(\hbar/2VK_\perp)I/e \propto I$.

Second, consider the case of *magnetic fan*.[19] Here the easy axis of the polarizer is again directed along $\hat{x}$, but the polarizer is perpendicular to the easy plane: $\vec{s} = (0,0,1)$, $\theta_s = 0$. This arrangement is known to produce a constant precession of vector $\vec{n}$. Eq. (9) gets a form:

$$\alpha\dot{\phi} = -\frac{\gamma K_{||}}{2M_s}\sin 2\phi - u \tag{11}$$

for $|u| < \gamma K_{||}/(2M_s)$ the current deflects the analyzer direction from the easy axis direction. For larger values of u there is no time-independent solution. The angles ϕ grows with time which corresponds to $\vec{n}$ making full rotations. At $|u| \gg \gamma K_{||}/(2M_s)$ the rotation frequency of the magnetic fan is given by $\omega \sim u/\alpha$.

III. DEVICE WITH TWO DYNAMIC MAGNETS (TWO "FREE LAYERS")

No let us assume that both magnets in Fig. 1A have finite size. Each magnet serves as a polarizer for the other one. Without approximations, the evolution of two sets of polar angles (θ_i, ϕ_i), $i = 1, 2$ is described by two LLG systems of equations

$$\begin{aligned}
\dot{\theta}^{(i)} + \alpha_i\dot{\phi}^{(i)}\sin\theta^{(i)} &= -\frac{\gamma}{M_{si}\sin\theta^{(i)}}\frac{\partial E^{(i)}}{\partial\phi^{(i)}} + \\
&+ u_{ji}(\vec{n}^{(j)}\cdot\vec{e}^{(i)}_\theta) \qquad (12)\\
\dot{\phi}^{(i)}\sin\theta^{(i)} - \alpha_i\dot{\theta}^{(i)} &= \frac{\gamma}{M_{si}}\frac{\partial E^{(i)}}{\partial\theta^{(i)}} + u_{ji}(\vec{n}^{(j)}\cdot\vec{e}^{(i)}_\phi)
\end{aligned}$$

where j means the index not equal to i and no summation is implied.

We now apply the overdamped, large easy-plane anisotropy approximation to both magnets. Equation (9)

for each magnet is further simplified since for the magnet i the angle $\theta_s = \theta_j = \pi/2 + \delta\theta_j$, $\delta\theta_j \ll 1$. Expanding (9) in small $\delta\theta_j$ and using the slave condition (6) for $\delta\theta_j$ with $(\vec{s}\cdot\vec{e}_\phi) = (\vec{n}^{(j)}\cdot\vec{e}^{(i)}_\phi)$ expanded in both small angles (see Eq. (A5)) we get the system:

$$(\alpha_i + 2\xi_{ji}\cos(\phi_j - \phi_i))\dot{\phi}_i - \\ -\xi_{ji}(\cos(\phi_j - \phi_i) + 1)\dot{\phi}_j = -\frac{\partial E^{(i)}}{\partial \phi_i}\ , \tag{13}$$

with $\xi_{ji} = u_{ji}M_{si}/(\gamma K_\perp)$. It was assumed that $K_\perp$ is the same for both magnets.

The spin-transfer torque parameters u_{21} and u_{12} have opposite signs and their absolute values are different due to different volumes of the magnets, according to Eq. (2). We assume $V_1 \geq V_2$ and denote $u_{12} = u$, $u_{21} = -\epsilon u$. The larger magnet experiences a relatively smaller spin transfer effect, and the asymmetry parameter satisfies $0 \leq \epsilon \leq 1$. In general, material parameters $\alpha_{1,2}$, $M_{s1,2}$ and magnetic anisotropy energies $E^{(1,2)}$ of the two magnets are also different, but here we focus solely on the asymmetry in spin-transfer parameters. Both $E^{(1)}$ and $E^{(2)}$ are assumed to be given by formula (4) with the same direction of in-plane easy axis. The situation can be viewed as a collinear switching setup with dynamic polarizer. Equations (13) specialize to

$$\left|\begin{array}{cc} \alpha - 2\epsilon\xi C & \epsilon\xi(C+1) \\ -\xi(C+1) & \alpha + 2\xi C \end{array}\right| \left[\begin{array}{c} \dot{\phi}_1 \\ \dot{\phi}_2 \end{array}\right] = -\frac{\omega_0}{2}\left[\begin{array}{c} \sin 2\phi_1 \\ \sin 2\phi_2 \end{array}\right]$$

$$C = \cos(\phi_1 - \phi_2), \quad \omega_0 = \frac{\gamma K_{||}}{M_s} \tag{14}$$

Next, we study the stability of all equilibrium configurations (ϕ_1, ϕ_2) of two magnets. There are four equilibrium states that are stable without the current: two parallel states along the easy axis $(0,0)$ and (π,π), two antiparallel states along the easy axis $(0,\pi)$ and $(\pi,0)$. Four more equilibrium states have magnetization perpendicular to the easy axis and are unstable without the current: $(\pm\pi/2, \pm\pi/2)$. Once again, since spin-transfer does not depend on the relative direction of current and magnetization, the configurations which can be transformed into each other by a rotation of the magnetic space as a whole behave identically. Thus it is enough to consider four configurations: $(0,0)$, $(0,\pi)$, $(\pi/2,\pi/2)$, and $(\pi/2,-\pi/2)$. We linearize equations (14) near each equilibrium and search for the solution in the form $\delta\phi_i \sim \exp(\omega t)$. The eigenfrequencies are found to be:

$$\begin{aligned}
(0,0):& \quad \omega_1 = \frac{-\omega_0}{\alpha}, \quad \omega_2 = -\frac{-\omega_0}{\alpha + 2\xi(1-\epsilon)} \\
(0,\pi):& \quad \omega_1 = \frac{-\omega_0}{\alpha + 2\epsilon\xi}, \quad \omega_2 = \frac{-\omega_0}{\alpha - 2\xi} \\
(\frac{\pi}{2},\frac{\pi}{2}):& \quad \omega_1 = \frac{\omega_0}{\alpha}, \quad \omega_2 = \frac{\omega_0}{\alpha - 2\xi(1-\epsilon)} \\
(\frac{\pi}{2},-\frac{\pi}{2}):& \quad \omega_1 = \frac{\omega_0}{\alpha + 2\epsilon\xi}, \quad \omega_2 = \frac{\omega_0}{\alpha - 2\xi}
\end{aligned}$$

The state is stable when both eigenfrequencies are negative. We conclude that initially unstable states $(\pi/2, \pm\pi/2)$ are never stabilized by the current, while the $(0,0)$ and $(0,\pi)$ state remain stable for

$$\begin{aligned}
(0,0):& \quad \xi > -\frac{\alpha}{2(1-\epsilon)} \\
(0,\pi):& \quad -\frac{\alpha}{2\epsilon} < \xi < \frac{\alpha}{2}
\end{aligned}$$

These regions of stability are shown schematically in Fig. 2B in comparison with the case of static magnetic polarizer (Fig. 2A) which is recovered at $\epsilon \to 0$.

As the size of the polarizer is reduced, the asymmetry parameter ϵ grows. The stability region of the antiparallel state acquires a lower boundary $\xi = -\alpha/(2\epsilon)$. Up to $\epsilon = 1/2$, this boundary is still below the lower boundary of the parallel configuration stability region. Consequently, the parallel configuration is switched to the antiparallel at a negative current $\xi = -\alpha/(2(1-\epsilon))$. The system then remains in the antiparallel state down to $\xi = -\alpha/(2\epsilon)$. Below that threshold no stable configurations exist, and the system goes into some type of precession state. This dynamic state is related to the "windmill" state predicted in Ref. 2 for two identical magnets in the absence of anisotropies. Obviously, here it is modified by the strong easy-plane anisotropy.

The $\epsilon = 1/2$ value represents a transition point in the behavior of the system. For $1/2 < \epsilon < 1$, the stability region of the parallel configuration completely covers the one of the antiparallel state. A transition without hysteresis now happens at $\xi = -\alpha/(2(1-\epsilon))$ between the parallel state and the precession state. If the system is initially in the antiparallel state, it switches to the parallel state either at a negative current $\xi = -\alpha/(2(1-\epsilon))$ or at a positive current $\xi = \alpha/2$, and never returns to the antiparallel state after that.

IV. CONCLUDING REMARKS

We studied the behavior of planar spin-transfer devices with magnetic energy dominated by the large easy-plane anisotropy. The overdamped approximation in the presence of current-induced torque was derived and checked against the cases already discussed in the literature. In the new "dynamic polarizer" case, we found a transition between two regimes with different switching sequences. The large asymmetry regime is similar to the case of static polarizer and shows hysteretic switching between the parallel and antiparallel configurations, while in the small asymmetry regime the magnets do not switch, but go directly into the "windmill" precession state.

We saw that the current-induced switching occurs when the effective damping constant vanishes near a particular equilibrium. This makes the overdamped approximation inapplicable in the immediate vicinity of the transition and renders Eqs. (14) ill-defined at some points. However, the overall conclusions about the switching

events will remain the same as long as the interval of inapplicability is small enough.

We also find that the overdamped planar approximation does not work well when a saddle point of magnetic energy is stabilized by spin-transfer torque, e.g. during the operation of a spin-flip transistor.[20] Description of such cases in terms of effective planar equations requires additional investigations.

V. ACKNOWLEDGEMENTS

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APPENDIX A: VECTOR DEFINITIONS

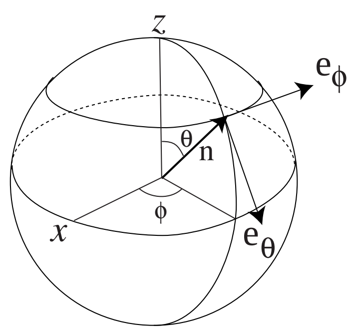

FIG. 3: Definitions of the tangent vectors and polar angles.

We use the standard definitions of polar coordinates and tangent vectors (see Fig. 3):

$$\begin{aligned} \vec{n} &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\ \vec{e}_\theta &= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\ \vec{e}_\phi &= (-\sin\phi, \cos\phi, 0) \end{aligned} \tag{A1}$$

When $\theta = \pi/2 + \delta\theta$ a linearization in $\delta\theta$ gives

$$\begin{aligned} \vec{n} &\approx (\cos\phi, \sin\phi, -\delta\theta) \\ \vec{e}_\theta &\approx -(\delta\theta\cos\phi, \delta\theta\sin\phi, 1) \\ \vec{e}_\phi &\approx (-\sin\phi, \cos\phi, 0) \end{aligned} \tag{A2}$$

For two unit vectors $\vec{n}^{(i)}$, $i = 1, 2$ with polar angles (θ_i, ϕ_i) the scalar product expressions are

$$\begin{aligned} (\vec{n}^{(j)} \cdot \vec{e}_\theta^{(i)}) &= \sin\theta_j\cos\theta_i\cos(\phi_j - \phi_i) - \cos\theta_j\sin\theta_i \\ (\vec{n}^{(j)} \cdot \vec{e}_\phi^{(i)}) &= \sin\theta_j\sin(\phi_j - \phi_i) \end{aligned} \tag{A3}$$

Linearizing (A3) with respect to small $\delta\theta_i$ for arbitrary values of θ_j one gets:

$$\begin{aligned} (\vec{n}^{(j)} \cdot \vec{e}_\theta^{(i)}) &\approx -\sin\theta_j\delta\theta_i\cos(\phi_j - \phi_i) - \cos\theta_j \\ (\vec{n}^{(j)} \cdot \vec{e}_\phi^{(i)}) &\approx \sin\theta_j\sin(\phi_j - \phi_i) \end{aligned} \tag{A4}$$

Linearization of (A3) with respect to both $\delta\theta_i$ and $\delta\theta_j$ gives

$$\begin{aligned} (\vec{n}^{(j)} \cdot \vec{e}_\theta^{(i)}) &\approx -\delta\theta_i\cos(\phi_j - \phi_i) + \delta\theta_j \\ (\vec{n}^{(j)} \cdot \vec{e}_\phi^{(i)}) &\approx \sin(\phi_j - \phi_i) \end{aligned} \tag{A5}$$

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